A Model of Capital and Crises

Zhiguo He
Booth School of Business, University of Chicago

Arvind Krishnamurthy
Northwestern University and NBER

AFA, 2011
Introduction

- Intermediary capital can affect asset prices.
- We study the role of distressed intermediary capital in the crises of market with complex asset classes (e.g. MBS).
Introduction

- Intermediary capital can affect asset prices.
- We study the role of distressed intermediary capital in the crises of market with complex asset classes (e.g. MBS).
- A General Equilibrium (GE) model where intermediaries, rather than households, are marginal.
  - Frictions are endogenously derived based on optimal contracting considerations.
- Mechanism: Intermediation capital affects participation/risk-sharing.
- In normal times households participate through intermediation;
- When intermediaries suffer losses,
  - Distressed intermediary sector averse to hold risky positions, risk premium goes up.
  - Households “fly to quality,” drive down interest rate.
Model Structure (1)

- Unit supply of **risky asset** with dividend $\frac{dD_t}{D_t} = gdt + \sigma dZ_t$, and **riskless asset** in zero-net supply.
  - Risky asset price $P_t$ and interest rate $r_t$ are determined in GE.

- **Households** $\mathbb{E} \left[ \int_0^\infty e^{-\rho^h t} \ln c^h_t \, dt \right]$.
  - Limited participation in risky asset market. They invest in intermediaries.

- **Specialists** $\mathbb{E} \left[ \int_0^\infty e^{-\rho t} \ln c_t \, dt \right]$, $\rho < \rho^h$. They run **intermediaries**.
  - Only intermediaries/specialists can invest in the risky asset. They are marginal investors.
  - Derive **Intermediation Constraint** from moral hazard primitives.
Model Structure (2)

- **Intermediation**: 1) Short-term contracting between agents; 2) Equilibrium in competitive intermediation market;
- **Asset pricing**: 3) Optimal consumption/portfolio decisions; 4) GE.
The Heart of the Model: (equity) Capital Constraint

- Say household with wealth $W_t^h$, and specialist with wealth $W_t$.
  - Given specialist’s contribution $W_t$ in the intermediary, household contributes $T_t^h$ as equity investment (for risk sharing).
  - Capital Constraint: $T_t^h$ is capped at $mW_t$ so risk sharing is capped at $1 : m$.

- Intermediation capacity $mW_t$ is increasing in the specialist’s contribution $W_t$, as reflection of agency friction.
The Heart of the Model: (equity) Capital Constraint

- Say household with wealth $W_t^h$, and specialist with wealth $W_t$.
  - Given specialist's contribution $W_t$ in the intermediary, household contributes $T_t^h$ as equity investment (for risk sharing).
  - Capital Constraint: $T_t^h$ is capped at $mW_t$ so risk sharing is capped at $1 : m$.

- Intermediation capacity $mW_t$ is increasing in the specialist's contribution $W_t$, as reflection of agency friction.

- How to interpret $m$?
  1. Intermediary capital requirement: outside/inside contribution ratio; (Holmstrom-Tirole, QJE)
     - Officers/Directors inside holdings in financial industry around 18%.
  2. Incentive contract—the performance share of hedge fund managers. Think of “2 and 20.”
  3. Mutual funds’ flow-performance sensitivity. Specialist’s $W_t$ tracks his past gains and losses (Shleifer-Vishny, JF)
Intermediation Constraint: An Example (1)

- Say $m = 1$, $W_t^h = 80$. Comparing $W_t^h$ to $mW_t$.
- **Unconstrained Region:** $W_t = 100$. Then $T_t^h = W_t^h = 80$;
  - Zero net debt. Risky asset price $P_t = W_t + W_t^h = 180$.
  - Fund’s total equity 180. Intermediary holds risky asset without leverage, first-best risk sharing.
**Constrained Region:**  \( W_t = 50 \). Then \( T^h_t = mW_t = 50 \);
Intermediation Constraint: An Example (2)

- **Constrained Region:** \( W_t = 50 \). Then \( T^h_t = mW_t = 50 \);
- Intermediary’s total equity is \( 50 + 50 = 100 \). But \( P_t = 130 \).
- In equilibrium, the intermediary borrows 30 from the debt market;
  - It is supplied by households \( W^h_t - T^h_t = 30 \).
- Specialist and household have equal shares in the intermediary;
- Specialist’s leveraged position in risky asset:
  \[ \alpha = \frac{\text{specialist’s portion of asset}}{\text{specialist’s equity}} = \frac{130}{50} = 130\% . \]
- Risk premium has to adjust to make this high leverage optimal.
Risk Premium and Interest Rate

Scaled Specialist's Wealth $w$ vs Risk Premium $w^c$ for $m=4$ and $m=6$

Interest Rate vs Scaled Specialist's Wealth $w$ for $m=4$ and $m=6$

$w^c(m=4) = 13.02$ for $m=4$

$w^c(m=6) = 9.07$ for $m=6$
Intermediation Stage Game

- **Short-term** contracts only. At time $t$, contract from $t$ to $t + dt$.
- Household with wealth $W^h_t$, and specialist with wealth $W_t$.
  - Household contributes $T^h_t$, specialist $T_t$. $T^l_t = T^h_t + T_t$. 
- **Moral Hazard:**
  - Unobserved due diligence action $s_t = f_0, 1$.
  - Shirking ($s_t = 1$) reduce return by $X_t$ but brings private benefit $B_t$.
  - Unobserved portfolio choice $E_{It}$ (dollar exposure to risky asset); undoing activity. Not crucial.
- Fund's return $E_{It} (dR_t r_t dt) + T_{It} r_t dt s_t X_t dt$, private benefits $s_t B_t dt$. Focus on implementing working.
Intermediation Stage Game

- **Short-term** contracts only. At time \( t \), contract from \( t \) to \( t + dt \).
- Household with wealth \( W^h_t \), and specialist with wealth \( W_t \).
  - Household contributes \( T^h_t \), specialist \( T_t \). \( T_t = T^h_t + T_t \).
- Specialist in charge of intermediary. **Moral Hazard:**
  1. Unobserved **due diligence action** \( s_t = \{0, 1\} \).
     - Shirking \( (s_t = 1) \) reduce return by \( X_t \) but brings private benefit \( B_t \).
  2. Unobserved **portfolio choice** \( E^l_t \) (dollar exposure to risky asset);
     - Undoing activity. Not crucial.
- Fund’s return \( E^l_t (dR_t - r_t dt) + T^l_t r_t dt - s_t X_t dt \), private benefit \( s_t B_t dt \). Focus on implementing working.
Intermediation Contract

- **Affine contracts** for sharing returns.
  - $\beta_t$: specialist’s share; $\hat{K}_t dt$: transfer to specialist.
- Set $K_t \equiv \left( \beta_t T_t^l - T_t \right) r_t + \hat{K}_t$.
- Dynamic budget constraint
  \[
  \begin{cases}
  dW_t = r_t W_t dt - c_t dt + \beta_t \mathcal{E}_t^l (dR_t - r_t dt) + K_t dt, \\
  dW_t^h = r_t W_t^h dt - c_t^h dt + (1 - \beta_t) \mathcal{E}_t^l (dR_t - r_t dt) - K_t dt.
  \end{cases}
  \]
- Reduce contract to $(\beta_t, K_t)$. **Sharing rule** and fee.
  - Specialist chooses $\mathcal{E}_t = \beta_t \mathcal{E}_t^l$. Household buys risk exposure $\mathcal{E}_t^h = (1 - \beta_t) \mathcal{E}_t^l$ from intermediary.
  - In competitive intermediation market, the fee will take some simple linear form.
IC Constraint and Maximum Household’s Exposure

- **IC constraint**: specialist bears at least a certain fraction of risk.
  - Incentive provision. Skin in the game.
  - No shirking: $\beta_t X_t - B_t \geq 0 \Rightarrow \beta_t \geq \frac{B_t}{X_t} \equiv \frac{1}{1+m}$.
  - A lower bound on $\beta_t$.

- Specialist always chooses $\beta_t E_t^l = E_t^*$ independent of $\beta_t$.
  - In the paper we show $E_t^*$ is independent of $K$. 
IC Constraint and Maximum Household’s Exposure

- **IC constraint**: specialist bears at least a certain fraction of risk.
  - Incentive provision. Skin in the game.
  - No shirking: $\beta_t X_t - B_t \geq 0 \Rightarrow \beta_t \geq \frac{B_t}{X_t} \equiv \frac{1}{1+m}$.
  - A lower bound on $\beta_t$.

- Specialist always chooses $\beta_t E^l_t = E^*_t$ independent of $\beta_t$.
  - In the paper we show $E^*_t$ is independent of $K$.

- $E^l_t$ fund’s total risk exposure. S: $E_t = \beta_t E^l_t$, H: $E^h_t = (1 - \beta_t) E^l_t$.
  - Household exposure from the contract, or **exposure supply**:

  $$E^h_t = (1 - \beta_t) E^l_t = \frac{1 - \beta_t}{\beta_t} E^*_t.$$  

- As $\beta_t \geq \frac{1}{1+m}$, **households maximum exposure** $E^h_t \leq m E^*_t$. 
Key Intuition and Equity Implementation

- The households exposure is capped due to agency frictions
  \[ \mathcal{E}_t^h \leq m\mathcal{E}_t^*. \]

- It caps a risk-sharing rule between households and specialists.
  - Incentive provision implies that specialists have to bear sufficient risk.

- In bad times this friction kicks in.
  - Even if specialists wealth is low, they still have to bear disproportionately large risk.
Key Intuition and Equity Implementation

- The households exposure is capped due to agency frictions $\mathcal{E}_t^h \leq m\mathcal{E}_t^*$. 
- It caps a risk-sharing rule between households and specialists.
  - Incentive provision implies that specialists have to bear sufficient risk.
- In bad times this friction kicks in.
  - Even if specialists wealth is low, they still have to bear disproportionally large risk.

- **Equity implementation**: Households (outsiders) cannot hold more than $\frac{m}{1+m}$ (equity) shares.
- **Equity capital constraint**: Given specialist’s equity $W_t$, households can make at most $mW_t$ equity contributions.
- Recall contract $(\beta_t, K_t)$. We have derived equilibrium $\beta_t$. What determines fee $K_t$?
  - Households pay competitive fees in the intermediation market.
Competitive Intermediation Market

At time $t$, specialists make offers $(\beta_t, K_t)$ to households who can accept/reject offers. The intermediation market reaches equilibrium if: 1) $\beta_t$ is incentive compatible; 2) no profitable deviation coalitions.

Lemma: In equilibrium, households face a per-unit-exposure price of $k_t \geq 0$: to purchase $\mathcal{E}_t^h$, he has to pay $K_t = k_t \mathcal{E}_t^h$.

- Idea: equivalence between core and Walrasian equilibrium.
  - Households and specialists form coalitions to chop off the exposure linearly.
- Now we start studying agents’ consumption/portfolio problems.
Households’ Consumption/Portfolio Rules

- Log investors. Simple consumption rule; myopic mean-variance portfolio choice.

- Risky asset excess return
  \[ dR_t - r_t \, dt = \pi_{R,t} \, dt + \sigma_{R,t} \, dZ_t. \]

- Household
  \[
  \max_{\{c_t, \mathcal{E}_t\}} \mathbb{E} \left[ \int_0^\infty e^{-\rho^h t} \ln c^h_t \, dt \right]
  \]
  subject to
  \[
  dW^h_t = W^h_t \, r_t \, dt - c^h_t \, dt + \mathcal{E}^h_t (dR_t - r_t \, dt) - k_t \mathcal{E}^h_t \, dt.
  \]

- Standard problem; households achieve exposure \( \mathcal{E}_t^h \) by paying per-unit-cost of \( k_t \).

- Optimal consumption
  \[
  c_t^{h*} = \rho^h W_t^h,
  \]
  optimal exposure
  \[
  \mathcal{E}_t^{h*} = \frac{\pi_{R,t} - k_t}{\sigma^2_{R,t}} W_t^h.
  \]
  Optimal risk exposure is decreasing in exposure price \( k_t \).
Specialists’ Consumption/Portfolio Rules

- The specialist supplies an exposure \( \frac{1-\beta_t}{\beta_t} \mathcal{E}_t^* \). Given exposure price \( k_t \), he gets intermediation fees \( K_t dt = k_t \left( \frac{1-\beta_t}{\beta_t} \mathcal{E}_t^* \right) dt \).

- The specialist is solving: \( \max_{\{c_t, \mathcal{E}_t, \beta_t\}} \mathbb{E} \left[ \int_0^{\infty} e^{-\rho t} \ln c_t dt \right] \) subject to

\[
dW_t = \mathcal{E}_t (dR_t - r_t dt) + \max_{\beta_t \in \left[ \frac{1}{1+m}, 1 \right]} \left( \frac{1 - \beta_t}{\beta_t} \right) k_t \mathcal{E}_t^* dt + W_t r_t dt - c_t dt.
\]

- \( \beta_t^* = \frac{1}{1+m} \) if \( k_t > 0 \), otherwise \( \beta_t^* \in \left[ \frac{1}{1+m}, 1 \right] \) if \( k_t = 0 \). Exposure supply schedule.

- \( \mathcal{E}_t^* \) is the exposure expected by households, and must coincide with the specialist’s actual optimal choice in REE.
Specialists’ Consumption/Portfolio Rules

- The specialist supplies an exposure \( \frac{1-\beta_t}{\beta_t} \mathcal{E}_t^* \). Given exposure price \( k_t \), he gets intermediation fees \( K_t \, dt = k_t \left( \frac{1-\beta_t}{\beta_t} \mathcal{E}_t^* \right) \, dt \).

- The specialist is solving: \( \max \mathbb{E} \left[ \int_0^\infty e^{-\rho t} \ln c_t \, dt \right] \) subject to

\[
dW_t = \mathcal{E}_t \left( dR_t - r_t \, dt \right) + \max_{\beta_t \in [\frac{1}{1+m}, 1]} \left( \frac{1 - \beta_t}{\beta_t} \right) k_t \mathcal{E}_t^* \, dt + W_t r_t \, dt - c_t \, dt.
\]

- \( \beta_t^* = \frac{1}{1+m} \) if \( k_t > 0 \), otherwise \( \beta_t^* \in \left[ \frac{1}{1+m}, 1 \right] \) if \( k_t = 0 \). Exposure supply schedule.

- \( \mathcal{E}_t^* \) is the exposure expected by households, and must coincide with the specialist’s actual optimal choice in REE.

- Solution: \( c_t^* = \rho W_t \) and \( \mathcal{E}_t^* = \frac{\pi_{R,t}}{\sigma_{R,t}^2} W_t \), and specialists receive fee of \( K_t = q_t W_t \) where \( q_t = \left( \frac{1-\beta_t^*}{\beta_t^*} \right) k_t \frac{\pi_{R,t}}{\sigma_{R,t}^2} \).
Unconstrained vs. Constrained Regions (1)

Unconstrained Region

Exposure demand

\[ \left( \frac{\pi_{R_t} - k_t}{\sigma^2_{R_t}} \right) W_t^k \]

Exposure supply

\[
\begin{cases}
0, m \left( \frac{\pi_{R_t}}{\sigma^2_{R_t}} \right) W_t & \text{if } k_t = 0, \\
m \left( \frac{\pi_{R_t}}{\sigma^2_{R_t}} \right) W_t & \text{if } k_t > 0.
\end{cases}
\]
Constrained Region

Exposure supply

\[
\begin{cases}
0, m \left( \frac{\pi_R}{\sigma_R^2} \right) W_t & \text{if } k_t = 0, \\
m \left( \frac{\pi_R}{\sigma_R^2} \right) W_t & \text{if } k_t > 0.
\end{cases}
\]

Exposure demand

\[
\left( \frac{\pi_{R,t} - k_t}{\sigma_{R,t}^2} \right) W_t^k
\]
Equilibrium Asset Prices: Solution

- We derive everything in closed form.
- State variables \((D_t, W_t)\). Scales with \(D_t\).
- Uni-dimensional state variable \(w_t \equiv W_t / D_t\) captures wealth distribution.
Equilibrium Asset Prices: Solution

- We derive everything in closed form.
- State variables \((D_t, W_t)\). Scales with \(D_t\).
- Uni-dimensional state variable \(w_t \equiv W_t / D_t\) captures wealth distribution.
- Consumption rules \(c^*_t = \rho W^h_t\), \(c^{h*}_t = \rho^h W^h_t\).
- Zero net debt \(W_t + W^h_t = P_t\), goods clearing \(c^*_t + c^{h*}_t = D_t\). So
  \[
  \frac{P_t}{D_t} = \frac{1}{\rho^h} + \left(1 - \frac{\rho}{\rho^h}\right) w_t.
  \]
- Specialist’s risky (percentage) position \(\alpha_t = \frac{P_t}{(1+m)W_t} > 1\) in constrained region.
The economy is in constrained region whenever

\[ w_t = W_t / D_t < w^c \equiv \frac{1}{m\rho_h + \rho}. \]

In unconstrained region, \( w_t \) increases deterministically toward \( w^c \).
- Perfect risk sharing rule. Relative patience level \( \rho < \rho^h \) matters.

In constrained region, specialists take a higher leverage than households. Therefore \( w_t \) becomes stochastic and drops when fundamental falls.

When (scaled) intermediary capital \( w_t \) falls in constrained region,
- Risk premium rises;
- Interest rate falls;
- Volatility rises;
- Correlation endogenously rises.
<table>
<thead>
<tr>
<th></th>
<th>Uncon. Region</th>
<th>Con. Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^*_t$</td>
<td>$W_t$</td>
<td>$\frac{1}{1+m} P_t$</td>
</tr>
<tr>
<td>$\alpha_t$</td>
<td>1</td>
<td>$\frac{1+(\rho^h-\rho)w_t}{(1+m)\rho^h w_t} &gt; 1$</td>
</tr>
<tr>
<td>$\sigma_{R,t}$</td>
<td>$\sigma$</td>
<td>$\frac{\sigma}{1+(\rho^h-\rho)w_t} \left( \frac{(1+m)\rho^h}{m\rho^h+\rho} \right) &gt; \sigma$</td>
</tr>
<tr>
<td>$\pi_{R,t}$</td>
<td>$\sigma^2$</td>
<td>$\frac{\sigma^2}{w_t(m\rho^h+\rho)} \left( \frac{(1+m)\rho^h}{m\rho^h+\rho} \right) \left( \frac{1}{1+(\rho^h-\rho)w_t} \right) &gt; \sigma^2$</td>
</tr>
<tr>
<td>$k_t$</td>
<td>0</td>
<td>$\frac{1-(\rho+m\rho^h)w_t}{(1-\rho w_t)(1+(\rho^h-\rho)w_t)} \frac{(1+m)\rho^h \sigma^2}{w_t(m\rho^h+\rho)^2} &gt; 0$</td>
</tr>
<tr>
<td>$r_t$</td>
<td>$\rho^h + g - \sigma^2$</td>
<td>$-\sigma^2 \left[ \rho \left( (1+m) \frac{1}{w_t} - \rho - m^2 \rho^h \right) - (m\rho^h)^2 \right] \left( 1-\rho w_t \right) \left( \rho + m\rho^h \right)^2$</td>
</tr>
</tbody>
</table>
Asymmetry. Crisis like.

When constraint binds \( w_t < w^c \), specialist bears disproportionally large risk, causing more volatile pricing kernel.

Flight to quality. 1) Specialists precautionary savings. 2) Household fly to debt market.
Comovement

- Consider an infinitesimal asset with

\[
\frac{d\hat{D}_t}{\hat{D}_t} = \frac{dD_t}{D_t} + \hat{\sigma} d\hat{Z}_t.
\]

- The correlation between \(dR_t\) and \(\hat{R}_t\) is:

\[
corr(dR_t, \hat{R}_t) = \frac{1}{\sqrt{1 + (\hat{\sigma} / \sigma_{R,t})^2}}.
\]

- Unconstrained region, since \(\sigma_R\) is constant, the correlation is constant.

- Constrained region, rising correlation.
  - Market return volatility \(\sigma_{R,t}\) rises, magnifying the common component of returns.
Concluding Remarks (1)

- Canonical intermediation friction meets canonical GE asset pricing models.
- Calibratable, easy to quantify effects.
- We have another paper where specialists have general CRRA power utility, with capital constraint as given.
  - Calibrate the model to the MBS market;
  - Add in labor income, debt households (create leverage in unconstrained region), and other necessary twists...
  - Study the crisis dynamics (especially recovery), government liquidity injection policies, etc.
Concluding Remarks (2): Calibration result

Crisis Recovery

<table>
<thead>
<tr>
<th>Transit from 12%</th>
<th>W/o Capital Infusion</th>
<th>W. Capital Infusion (48bn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>0.17</td>
<td>0</td>
</tr>
<tr>
<td>7.5%</td>
<td>0.66</td>
<td>0.31</td>
</tr>
<tr>
<td>5%</td>
<td>2.72</td>
<td>2.20</td>
</tr>
<tr>
<td>4%</td>
<td>5.88</td>
<td>5.06</td>
</tr>
</tbody>
</table>