A Macroeconomic Framework for Quantifying Systemic Risk

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Arvind Krishnamurthy, Northwestern University and NBER

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Financial Crisis in the Model

Note: Capital constraint binds for $e < 0.435$
Note: The model does poorly on many standard macro calibration targets (e.g., no labor)

Model does well in capturing non-linearity in a select set of economic measures

... We will have to argue that our metric is a good one
Systemic Risk: What is the probability of the crisis in early 2007?

He and Krishnamurthy (Chicago, Northwestern)
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Based on initial condition chosen to match early 2007 asset prices:
- 1 year: 0.32%
- 2 year: 3.57%
- 5 year: 17.30%

Initial condition + rational forward looking agents = cant see around corners!
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Initial condition + rational forward looking agents = can't see around corners!

Stress test: Suppose we assume that roughly $2 trillion of shadow banking, with close to 0% capital, was not known to agents
- 1 year: 6.73%
- 2 year: 23.45%
- 5 year: 57.95%
Nonlinear macro model of a financial crisis

- Recent work on financial intermediaries: He-Krishnamurthy, Brunnermeier-Sannikov, Rampini-Viswanathan, Adrian-Boyarchenko, Gertler-Kiyotaki
- Our approach: occasionally binding constraint; global solution method (similar to Brunnermeier-Sannikov, Adrian-Boyarchenko)
Outline of Presentation

1. Nonlinear macro model of a financial crisis
   - Recent work on financial intermediaries: He-Krishnamurthy, Brunnermeier-Sannikov, Rampini-Viswanathan, Adrian-Boyarchenko, Gertler-Kiyotaki
   - Our approach: occasionally binding constraint; global solution method (similar to Brunnermeier-Sannikov, Adrian-Boyarchenko)

2. Calibration and Data
   - Nonlinearity in model and data
   - Match conditional moments of the data, conditioning on negative (i.e., recession) states
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2 Calibration and Data
   - Nonlinearity in model and data
   - Match conditional moments of the data, conditioning on negative (i.e., recession) states

3 Quantify systemic risk
   - Systemic risk: the state where financial intermediation is widely disrupted to affect real activities severely
     - In the model, states where capital constraint binds, crisis state
   - What is the ex-ante (e.g., initial conditions of 2007Q2) likelihood of crisis states? (... low)
   - What makes the probability higher?
   - Economics of stress tests (as opposed to accounting of stress tests)
Agents and Technology

Two classes of agents: households and bankers

- Households:
  \[
  \mathbb{E} \left[ \int_0^\infty e^{-\rho t} \frac{1}{1 - \gamma} C_t^{1-\gamma} dt \right], \quad C_t = (c_t^y)^{1-\phi} \left(c_t^h\right)^{\phi}
  \]

Two types of capital: productive capital $K_t$ and housing capital $H$.

- Fixed supply of housing $H \equiv 1$
- Price of capital $q_t$ and price of housing $P_t$ determined in equilibrium
Agents and Technology

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- Production \( Y = AK_t \), with \( A \) being constant

- Fundamental shocks: stochastic capital quality shock \( dZ_t \). TFP shocks
  \[ \frac{dK_t}{K_t} = i_t \, dt - \delta \, dt + \sigma dZ_t \]
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  \[
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  \]

- Investment/Capital $i_t$, quadratic adjustment cost
  \[
  \Phi(i_t, K_t) = i_t K_t + \frac{\kappa}{2} (i_t - \delta)^2 K_t
  \]
  \[
  \max_{i_t} q_t i_t K_t - \Phi(i_t, K_t) \Rightarrow i_t = \delta + \frac{q_t - 1}{\kappa}
  \]
Aggregate Balance Sheet

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_t K_t$

Equity $E_t$

Housing $P_t H$

Debt $W_t - E_t$

Household Sector

Financial Wealth

$W_t = q_t K_t + P_t H$
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$(1 - \lambda)W_t$

$\lambda W_t = \text{"Liquid balances" benchmark capital structure}$
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**Intermediary Sector**

- Capital $q_t K_t$
- Housing $P_t H$

**Equity $E_t$**

- Debt $W_t - E_t$

Separation of ownership and control

Banker maximizes $E[ROE] - \frac{m}{2} \text{Var}[ROE]$

**Household Sector**

Financial Wealth

$$W_t = q_t K_t + P_t H$$

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<table>
<thead>
<tr>
<th>Capital $q_t k_t$</th>
<th>$\text{equity}_t$</th>
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<td>Housing $P_t h_t$</td>
<td>$\text{debt}_t$</td>
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Portfolio share in capital: $\alpha_k^t = \frac{q_t k_t}{\text{equity}_t}$

Portfolio share in housing: $\alpha_h^t = \frac{P_t h_t}{\text{equity}_t}$

Borrowing (no constraint): $\text{debt}_t = q_t k_t + P_t h_t - \text{equity}_t = (\alpha_k^t + \alpha_h^t - 1) \text{equity}_t$
Bank Choice of Portfolio and Leverage

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Return on bank equity ROE: $d\tilde{R}_t = \alpha^k_t dR^k_t + \alpha^h_t dR^h_t - (\alpha^k_t + \alpha^h_t - 1) r_t dt$

Banker (log preference) solves: $\max_{\alpha^k_t, \alpha^h_t} \mathbb{E}_t [d\tilde{R}_t - r_t dt] - \frac{m}{2} \text{Var}_t [d\tilde{R}_t]$
### Bank Choice of Portfolio and Leverage

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\[
\max_{\alpha^k_t, \alpha^h_t} \mathbb{E}_t [d\tilde{R}_t - r_t dt] - \frac{m}{2} \text{Var}_t [d\tilde{R}_t]; \ m \text{ parameter}
\]

**Properties**

- \((k, h)\) scales with \text{equity}
- \((k, h)\) increasing in \(\mathbb{E}_t [d\tilde{R}_t - r_t dt]\)
- \((k, h)\) decreasing in \(\text{Var}_t [d\tilde{R}_t]\)
Given $E_t$, the equilibrium portfolio shares are pinned down by GE.

But portfolio shares must also be optimally chosen by banks, pinning down prices:

$$\max_{\alpha^k_t, \alpha^h_t} \mathbb{E}_t[d \tilde{R}_t - r_t dt] - \frac{m}{2} \text{Var}_t[d \tilde{R}_t]$$

Asset prices affect real side through investment ($q_t$).
Equity Dynamics in GE

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_t K_t$

-10%

Housing $P_t H$

-10%

Equity $E_t$

-10% × Lev

Debt $W_t - E_t$

Banker maximizes $E[ROE] - \frac{m}{2} \text{Var}[ROE]$

Financial Wealth

$W_t = q_t K_t + P_t H$

$(1 - \lambda) W_t$

$\lambda W_t = "Liquid balances"$

Household Sector
Equity Constraint

Loans to Capital Producers $i_t$

Intermediary Sector

Capital $q_t K_t$

Equity $E_t$

Constraint: $E_t \leq \mathcal{E}_t$

Debt $W_t - E_t$

Aggregate bank reputation $\mathcal{E}_t$

$\frac{d\mathcal{E}_t}{\mathcal{E}_t} = m \times \text{ROE}$, ROE is endogenous

Housing $P_t H$

Banker maximizes $E[\text{ROE}] - \frac{m}{2} \text{Var}[\text{ROE}]$

Household Sector

Financial Wealth

$W_t = q_t K_t + P_t H$

No constraint

$\lambda W_t = "\text{Liquid balances}"$

He and Krishnamurthy (Chicago, Northwestern)
Intermediary Reputation

- Single bank has “reputation" (skill, etc.) $\epsilon_t$ linked to intermediary performance (constant $m$)
  \[
  \frac{d\epsilon_t}{\epsilon_t} = md\tilde{R}_t.
  \]

- Poor returns reduce reputation: Berk-Green, 04; flow-performance relationship, Warther 95; Chevalier-Ellison, 97
- Or, $\epsilon_t$ as banker’s “net worth" fluctuating with performance
  - Kiyotaki-Moore 97, He-Krishnamurthy 12, Brunnermeier-Sannikov 12

- Household invests a maximum of $\epsilon_t$ dollars of equity capital with this banker
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- $\mathcal{E}_t$: aggregate reputation. Aggregate dynamics of $\mathcal{E}_t$

$$\frac{d \mathcal{E}_t}{\mathcal{E}_t} = md \tilde{R}_t - \eta dt + d\psi_t$$

- Exogenous death rate $\eta$. Endogenous entry $d\psi_t > 0$ of new bankers in extreme bad states
Equity Capital Constraint

- Representative household with $W_t$, split between bonds (at least) $\lambda W_t$ and equity (at most) $(1 - \lambda) W_t$

- Benchmark capital structure: $\lambda W_t$ of Debt, $(1 - \lambda) W_t$ of Equity
  - if there is no capital constraint ($\mathcal{E}_t$ is infinite)...
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- Benchmark capital structure: $\lambda W_t$ of Debt, $(1 - \lambda) W_t$ of Equity
  - if there is no capital constraint ($\mathcal{E}_t$ is infinite)...
- Intermediary equity capital:

$$E_t = \min [\mathcal{E}_t, (1 - \lambda) W_t]$$

- Suppose a $-10\%$ shock to real estate and price of capital:
- $W_t \downarrow 10\%$ (Household wealth = aggregate wealth)
- Reputation: $\frac{d\mathcal{E}_t}{\mathcal{E}_t} = md\tilde{R}_t + \ldots$ Two forces make $\mathcal{E}_t \downarrow$ more than $10\%$:
  1. Return on equity = $d\tilde{R}_t < -10\%$: equity is levered claim on assets
  2. $m > 1$ in our calibration
Calibration: Baseline Parameters

<table>
<thead>
<tr>
<th>Parameter Choice</th>
<th>Targets (Unconditional)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Intermediation</strong></td>
<td></td>
</tr>
<tr>
<td>$m$ Performance sensitivity</td>
<td>2</td>
</tr>
<tr>
<td>$\lambda$ Debt ratio</td>
<td>0.67</td>
</tr>
<tr>
<td>$\eta$ Banker exit rate</td>
<td>13%</td>
</tr>
<tr>
<td>$\gamma$ Entry trigger</td>
<td>6.5</td>
</tr>
<tr>
<td>$\beta$ Entry cost</td>
<td>2.43</td>
</tr>
<tr>
<td><strong>Panel B: Technology</strong></td>
<td></td>
</tr>
<tr>
<td>$\sigma$ Capital quality shock</td>
<td>3%</td>
</tr>
<tr>
<td>$\delta$ Depreciation rate</td>
<td>10%</td>
</tr>
<tr>
<td>$\kappa$ Adjustment cost</td>
<td>3</td>
</tr>
<tr>
<td>$A$ Productivity</td>
<td>0.133</td>
</tr>
<tr>
<td><strong>Panel C: Others</strong></td>
<td></td>
</tr>
<tr>
<td>$\rho$ Time discount rate</td>
<td>2%</td>
</tr>
<tr>
<td>$\xi$ 1/EIS</td>
<td>0.15</td>
</tr>
<tr>
<td>$\phi$ Housing share</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Results(1): State variable is $e_t = \mathcal{E}_t/K_t$

- Sharpe ratio
- Interest rate
- $q(e)$, capital price
- Investment $I/K$

- Capital constraint binds for $e < 0.435$
• Capital constraint binds for $e < 0.435$

• Without the possibility of the capital constraint, all of these lines would be flat. Model dynamics would be i.i.d., with vol=3%
State-dependent Impulse Response: -1% Shock \( (\sigma dZ_t) \)
Steady State Distribution
Nonlinearities in Model and Data

Model:
- Distress states = worst 33% of realizations of $e \ (e < 1.27)$
- Compute conditional variances, covariances of intermediary equity growth with other key variables

Data:
- Distress states = worst 33% of realizations of (risk premium in) credit spread
  - We use Gilchrist-Zakrajsek (2011) Excess Bond Premium, which we convert to a Sharpe ratio
  - Excess Bond Premium: risk premium of corporate bonds, presumably reflects distress of financial sector
  - Similar results if using NBER recessions
- Compute conditional variances, covariances of intermediary equity growth with other key variables
## Matching State-Dependent Covariances

<table>
<thead>
<tr>
<th></th>
<th>Distress</th>
<th>Non Distress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Baseline</td>
</tr>
<tr>
<td>$vol (Eq)$</td>
<td>31.48%</td>
<td>34.45</td>
</tr>
<tr>
<td>$vol (I)$</td>
<td>8.05%</td>
<td>5.30</td>
</tr>
<tr>
<td>$vol (C)$</td>
<td>1.71%</td>
<td>3.54</td>
</tr>
<tr>
<td>$vol (EB)$</td>
<td>60.14%</td>
<td>74.20</td>
</tr>
<tr>
<td>$cov (Eq, I)$</td>
<td>1.31%</td>
<td>1.05</td>
</tr>
<tr>
<td>$cov (Eq, C)$</td>
<td>0.25%</td>
<td>-0.96</td>
</tr>
<tr>
<td>$cov (Eq, LP)$</td>
<td>4.06%</td>
<td>5.87</td>
</tr>
<tr>
<td>$cov (Eq, EB)$</td>
<td>-6.81%</td>
<td>-14.95</td>
</tr>
</tbody>
</table>

- **Note**: without the capital constraint, all volatilities would be 3%, and have no state dependence.

- **What we do badly on**: Output vol is locally $\sigma$ because $Y_t = AK_t$. Financial friction only affects split between I and C.
Matching the 2007-2009 Crisis

He and Krishnamurthy (Chicago, Northwestern)
Based on EBS classification, economy crossed the 33% boundary ($e = 1.27$) between 2007Q2 and 2007Q3. Assume $e = 1.27$ in 2007Q2.

Then choose $(Z_{t+1} - Z_t)$ shocks to match realized intermediary equity series.

<table>
<thead>
<tr>
<th>Year</th>
<th>07QIII</th>
<th>07QIV</th>
<th>08QI</th>
<th>08QII</th>
<th>08QIII</th>
<th>08QIV</th>
<th>09QI</th>
<th>09QII</th>
<th>09QIII</th>
<th>09QIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>-2.5%</td>
<td>-4.2</td>
<td>-1.1</td>
<td>-1.1</td>
<td>-0.7</td>
<td>-1.6</td>
<td>-1.8</td>
<td>-1.8</td>
<td>-0.9</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

- Total -15.5%. Capital constraint binds after 07Q4—systemic risk state
- In the model (data), land price falls by 50% (55%)
- In the model (data), investment falls by 23% (25%)
Systemic Risk: What is the probability of the 2007-2009 crisis?

- Based on EBS classification, we cross the 33% boundary ($e = 1.27$) between 2007Q2 and 2007Q3.

- What is the likelihood of the constraint binding ("systemic crisis") assuming $e = 1.27$ currently:
  - 0.32% in next 1 years
  - 3.57% in next 2 years
  - 17.30% in next 5 years

Small...
Stress testing: “Hidden" Leverage

- Financial sector aggregate leverage fixed at 3 in model
  - We measure across commercial banks, broker/dealers, hedge funds in 2007:
    - Assets = $15,703 billion; Liabilities = $10,545 billion
- Pushed to crisis boundary after a -7% shock. 3.57% prob. of crisis in next 2 years

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- Pushed to crisis boundary after a -7% shock. 3.57% prob. of crisis in next 2 years
- Hidden leverage:
  - ABCP (SIVs): $1,189 billion; Liabilities $1,189 billion
  - Repo (MMFs and Sec Lenders): $1,020 billion; Liabilities $1,000 billion (assumed 2% haircut)
- Hidden in sense that agents take as given price functions and returns at leverage=3
  - 1 year: 6.73%
  - 2 year: 23.45%
  - 5 year: 57.95%
Stress testing

Key step: Need to map from stress scenario into underlying shock, $dZ_t$.

- Say stress scenario $\Rightarrow -30\%$ Return on equity
- Naive partial eqbm: leverage of 3, $\sigma(Z_{t+0.25} - Z_t) = -30/3 = -10\%$.
- Feed in $-10\%$ shock into the model over one quarter.
- Result: Beginning at $e = 1.27$ in 2007Q2, economy is immediately moved into crisis region, $e < 0.435$
- Our model helps in figuring out the right shock $dZ_t$

In US stress tests, scenario was over 6 quarters. Feed in shocks quarter-by-quarter, over 6 quarters:

<table>
<thead>
<tr>
<th>Return on Equity</th>
<th>6 QTR Shocks</th>
<th>Prob(Crisis within next 2 years)</th>
</tr>
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<tbody>
<tr>
<td>-2%</td>
<td>-1.16%</td>
<td>5.25 %</td>
</tr>
<tr>
<td>-5</td>
<td>-2.53%</td>
<td>8.90</td>
</tr>
<tr>
<td>-10</td>
<td>-4.69%</td>
<td>22.88</td>
</tr>
<tr>
<td>-15</td>
<td>-6.71%</td>
<td>48.90</td>
</tr>
<tr>
<td>-30</td>
<td>-8.72%</td>
<td>100.00</td>
</tr>
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</table>
Map “stress test” into a shock to $e$. 
Conclusion

- We develop a fully stochastic model of a systemic crisis, with an equity capital constraint on the intermediary sector.
- The model quantitatively matches the differential comovements in distress and non-distress periods.
- Is able to replicate 2007/2008 period with only intermediary capital shocks.
- Offers a way of mapping macro-stress tests into probability of systemic states.
Equity series